

# Information Saturation and Inner-Extremal Regular Black Holes

An Effective Model for Quantum Network Bridges

Concept manuscript - theoretical framework

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**Manuscript status.** This document formulates an effective theoretical model. The model does not claim experimental evidence for traversable portals or for a complete theory of quantum gravity. Its central claim is more limited: information saturation can be used as an effective order parameter for non-singular black holes, preserving the Schwarzschild limit, replacing the central singularity by a finite core, and suppressing the classical inner-horizon instability through an inner-extremal transition condition.

## Abstract

We formulate an effective theoretical framework in which black-hole singularities are replaced by a bounded state of local information saturation. The model, referred to here as the *Quantum Network Bridge* (QNB) framework, begins from the premise that gravity in the weak-field limit can be interpreted as a gradient of holographic information density. By relating the entropic loading of mass on an enclosing holographic screen to the Bekenstein bound, one obtains a dimensionless compactness variable  $\chi = 2GM/(c^2 r)$ , from which the Newtonian potential, the Schwarzschild horizon, and the standard gravitational time-dilation factor follow. We then construct an effective spherically symmetric black-hole solution in which the Misner-Sharp mass obeys a saturating profile. The central density consequently remains finite, and the Schwarzschild singularity is replaced by a de-Sitter-like core. Since generic regular black holes are threatened by mass inflation at the inner horizon, we introduce an inner-extremal QNB transition condition,  $\kappa_- = 0$ , realized by a metric function with a cubic zero at the inner horizon. This classically suppresses the exponential blueshift that drives mass inflation. The model is compatible with a Page-curve-like information evolution when the QNB saturation layer acts as a natural island region. Finally, we discuss falsifiable consequences, including strongly suppressed deviations in ringdown frequencies, possible gravitational-wave echoes for non-zero reflectivity of the transition layer, and a bounce or new-spacetime-branch interpretation of the black-hole interior. The proposed model is not a completed theory of quantum gravity, but a concrete effective framework that combines singularity resolution, information conservation, and inner-horizon stabilization within a single mathematical structure.

**Keywords:** black holes; quantum gravity; holography; information saturation; regular black holes; inner horizon; mass inflation; Page curve; quantum information.

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## 1. Introduction

Classical general relativity predicts that sufficiently compact gravitational collapse leads to the formation of a black hole with an event horizon and, in the ideal Schwarzschild or Kerr description, a central singularity. Such singularities are usually not interpreted as physical objects. Rather, they are understood as indications that the classical theory is being applied beyond its domain of validity. At or near the Planck scale, the dynamics of spacetime should presumably be replaced or supplemented by a theory of quantum gravity.

At the same time, black holes reveal a deep relation between geometry, thermodynamics, and information. Bekenstein proposed that black holes carry an entropy proportional to their horizon area [1], and Hawking showed that quantum fields on a black-hole background produce thermal radiation with a temperature determined by the surface gravity [2]. These results make black holes central objects in the search for a connection between relativity, quantum mechanics, and information theory.

This article develops an effective model in which this connection is formalized through a central quantity: information saturation. The basic idea is that a black-hole horizon is not only a causal surface, but also a surface on which the storage capacity of a region, measured by a holographic information limit, is maximally loaded. In the weak-field limit, this idea must reproduce ordinary Newtonian gravity. In the strong-field limit, it should replace the singularity by a finite saturation core.

We call the framework *Quantum Network Bridge* (QNB). The name reflects three assumptions. First, spacetime is regarded as an effective geometric description of an underlying quantum-information network. Second, gravity is interpreted as the macroscopic response of that information distribution. Third, a black-hole interior, once local information saturation is reached, may evolve into a bounce or a new spacetime branch. This last interpretation is speculative and is not presented in this article as an established result.

The strongest claim of this article is more limited and more precise:

An effective information-saturation model can reproduce the Schwarzschild limit, replace the central singularity by a finite core, and suppress the classical inner-horizon instability through an inner-extremal QNB transition condition.

## 2. Relation to existing literature

The proposed model is not detached from existing theory. It combines several established research directions into a specific effective structure.

The first direction is black-hole thermodynamics. The Bekenstein-Hawking entropy suggests that the number of degrees of freedom of a black hole scales with area rather than volume [1, 2]. Bekenstein also formulated a universal upper bound on the ratio between entropy, energy, and the size of a physical system [3]. This bound provides the basis for the definition of a holographic saturation variable in the present article.

The second direction is holography and entanglement geometry. The Ryu-Takayanagi formula relates entanglement entropy in holographic theories to minimal surfaces in a higher-dimensional

gravitational theory [4]. Jacobson showed that the semiclassical Einstein equation can, under specific assumptions, be connected to entanglement equilibrium in small spacetime regions [5]. These results support the broader idea that geometry and entanglement are not independent.

The third direction is regular black-hole physics. Hayward constructed non-singular black-hole models in which the static core has finite density and pressure and becomes cosmological-constant-like at small radius [6]. Frolov, Markov, and Mukhanov studied scenarios in which the singularity is replaced by a bounded-curvature region that may be connected to a closed or semi-closed new world [7].

The fourth direction concerns the instability of inner horizons. Poisson and Israel showed that the inner horizon of charged or rotating black holes can undergo mass inflation because of infinite blueshift and counter-streaming fluxes [8]. Recent work indicates that regular black holes with non-zero surface gravity at the inner horizon are generically unstable, while an inner-extremal horizon with  $\kappa_- = 0$  can classically suppress exponential mass-inflation growth [9].

The fifth direction concerns information conservation. Page analyzed the behavior of subsystem entropy in a unitary quantum-mechanical system [10]. Modern island and replica-wormhole calculations show, in specific models, how the Page curve of Hawking radiation can be reproduced from semiclassical geometry [11, 12].

Finally, QNB is conceptually related to ER=EPR-type ideas, in which entanglement and Einstein-Rosen bridges are connected. Maldacena and Susskind proposed that certain entangled black-hole states can be geometrically interpreted as non-traversable wormhole configurations [13]. Traversable-wormhole models further show that negative averaged null energy can, under specific circumstances, make an Einstein-Rosen bridge traversable [14]. QNB uses these results as motivation, not as proof of traversable portals.

### 3. Postulates of the QNB framework

We formulate QNB as an effective theory. The model is not intended to be a complete UV-complete theory of quantum gravity. It rests on five postulates.

#### 3.1. Spacetime as an effective information state

The fundamental description is a quantum-information network with total Hilbert space

$$\mathcal{H} = \bigotimes_i \mathcal{H}_i. \quad (1)$$

The classical spacetime  $(\mathcal{M}, g_{\mu\nu})$  is a large-scale, coarse-grained description of correlations and entanglement in this state. Formally, we write

$$g_{\mu\nu} = F_{\mu\nu}[\rho], \quad (2)$$

where  $\rho$  is the underlying quantum state. QNB does not fully specify  $F_{\mu\nu}$ . In this article, the model works with effective variables that reproduce gravity in the classical limit.

### 3.2. Holographic information saturation

To every closed two-dimensional screen  $\Sigma$  we assign a dimensionless saturation variable

$$\chi[\Sigma] = \frac{4\ell_P^2 S_{\text{ind}}[\Sigma]}{A[\Sigma]}. \quad (3)$$

Here  $A[\Sigma]$  is the area of the screen,  $S_{\text{ind}}$  is the entropic loading induced by matter or energy, and

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad (4)$$

is the Planck length. The limiting value

$$\chi[\Sigma] = 1 \quad (5)$$

is interpreted as holographic saturation. In spherical symmetry, this limit coincides with horizon formation.

### 3.3. Local information density is bounded

In addition to the screen variable  $\chi$ , we introduce a local saturation degree

$$\sigma(r) = \frac{\rho_{\text{info}}(r)}{\rho_c}, \quad (6)$$

where  $\rho_c$  is a critical information or energy density. The model requires

$$0 \leq \sigma(r) \leq 1. \quad (7)$$

This bound replaces the classical divergence  $\rho \rightarrow \infty$  by a finite saturation state.

### 3.4. General relativity as the low-saturation limit

When  $\sigma \ll 1$  and  $\chi \ll 1$ , the model must reproduce ordinary Newtonian gravity and general relativity. QNB may deviate measurably only in regimes of extreme compactness or information saturation.

### 3.5. The inner horizon is not a classical Cauchy horizon

A physically admissible QNB black hole may not contain a generic inner horizon with non-zero surface gravity. We therefore impose

$$\kappa_- = 0 \quad (8)$$

or, more strongly, require the inner horizon to be replaced by a finite quantum-information transition layer. In this article, the classical realization is given by a metric function with a cubic zero at  $r = r_-$ .

## 4. Newtonian gravity from holographic information saturation

The first criterion for any emergent-gravity model is that it reproduce ordinary Newtonian gravity.

Consider a spherical holographic screen of radius  $r$  surrounding a mass  $M$ . The Bekenstein bound relates the entropy of a system with energy  $E$  and radius  $R$  to

$$S \leq \frac{2\pi ER}{\hbar c}. \quad (9)$$

In QNB, we take as an effective weak-field rule that the entropic loading induced by a mass  $M$  on an enclosing screen is given by the saturating value

$$S_{\text{ind}}(r) = \frac{2\pi Er}{\hbar c} = \frac{2\pi Mcr}{\hbar}. \quad (10)$$

The screen area is

$$A = 4\pi r^2. \quad (11)$$

Hence

$$\chi(r) = \frac{4\ell_P^2 S_{\text{ind}}(r)}{A} \quad (12)$$

$$= \frac{4\ell_P^2}{4\pi r^2} \frac{2\pi Mcr}{\hbar}. \quad (13)$$

Using

$$\ell_P^2 = \frac{\hbar G}{c^3}, \quad (14)$$

one obtains

$$\chi(r) = \frac{2GM}{c^2 r}. \quad (15)$$

Define the Schwarzschild radius

$$r_s = \frac{2GM}{c^2}. \quad (16)$$

Then

$$\chi(r) = \frac{r_s}{r}. \quad (17)$$

This is the first central QNB relation. The classical horizon condition  $r = r_s$  becomes equivalent to  $\chi = 1$ . A black-hole horizon is thereby interpreted as a holographic saturation surface.

To reproduce the Newtonian potential, define

$$\Phi(r) = -\frac{c^2}{2}\chi(r). \quad (18)$$

Substitution gives

$$\Phi(r) = -\frac{GM}{r}. \quad (19)$$

The acceleration of a test mass follows from

$$\vec{a} = -\nabla\Phi, \quad (20)$$

and therefore

$$\vec{a} = -\frac{GM}{r^2}\hat{r}. \quad (21)$$

For a test mass  $m$ ,

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}. \quad (22)$$

For a general mass density  $\rho(\vec{x})$ , the natural generalization is

$$\chi(\vec{x}) = \frac{2G}{c^2} \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'. \quad (23)$$

Since

$$\nabla^2 \left( \frac{1}{|\vec{x} - \vec{x}'|} \right) = -4\pi\delta^3(\vec{x} - \vec{x}'), \quad (24)$$

it follows that

$$\nabla^2\chi = -\frac{8\pi G}{c^2}\rho. \quad (25)$$

This again yields the Poisson equation

$$\nabla^2\Phi = 4\pi G\rho. \quad (26)$$

QNB therefore reproduces Newtonian gravity when the information-saturation variable  $\chi$  is coupled to the Bekenstein entropic loading of mass on a holographic screen.

## 5. Time dilation and the Schwarzschild limit

In the weak-field limit of general relativity,

$$g_{00} \approx -\left(1 + \frac{2\Phi}{c^2}\right). \quad (27)$$

With  $\Phi = -c^2\chi/2$ , this becomes

$$g_{00} \approx -(1 - \chi). \quad (28)$$

For a static observer, the local time-dilation factor is

$$\frac{d\tau}{dt} = \sqrt{1 - \chi}. \quad (29)$$

For a point mass,

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2GM}{c^2 r}}. \quad (30)$$

This is the Schwarzschild time-dilation factor for a static observer outside a spherically symmetric mass.

The corresponding spherically symmetric metric is

$$ds^2 = -(1 - \chi(r))c^2 dt^2 + \frac{dr^2}{1 - \chi(r)} + r^2 d\Omega^2. \quad (31)$$

With  $\chi(r) = 2GM/(c^2r)$ , this becomes

$$ds^2 = - \left(1 - \frac{2GM}{c^2r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2r}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (32)$$

the Schwarzschild solution outside a spherical mass. Thus QNB reproduces not only Newtonian gravity, but also the correct static Schwarzschild limit.

## 6. Effective covariant form

The QNB model is not presented here as a fundamental action for quantum gravity. It can nevertheless be written covariantly as an effective correction to the Einstein equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + C_{\mu\nu}^{\text{QNB}}. \quad (33)$$

Here one must have

$$C_{\mu\nu}^{\text{QNB}} \rightarrow 0 \quad \text{for} \quad \sigma \ll 1. \quad (34)$$

At information saturation,

$$C_{\mu\nu}^{\text{QNB}} \neq 0. \quad (35)$$

The correction tensor must satisfy the covariant conservation condition

$$\nabla^\mu \left( T_{\mu\nu} + \frac{c^4}{8\pi G} C_{\mu\nu}^{\text{QNB}} \right) = 0. \quad (36)$$

If ordinary matter is locally conserved,  $\nabla^\mu T_{\mu\nu} = 0$ , then

$$\nabla^\mu C_{\mu\nu}^{\text{QNB}} = 0. \quad (37)$$

Therefore  $C_{\mu\nu}^{\text{QNB}}$  cannot be chosen arbitrarily. In a complete theory, it should follow from a diffeomorphism-invariant action. For the effective spherical model below,  $C_{\mu\nu}^{\text{QNB}}$  is equivalently described by an effective anisotropic stress-energy tensor that supports the regular core.

## 7. Regular black holes from a saturating mass function

In this section we work in geometrized units,  $G = c = 1$ . The factors  $G$  and  $c$  may be restored through  $r_s = 2GM/c^2$ .

Take the static spherically symmetric ansatz

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (38)$$

with

$$f(r) = 1 - \frac{2m(r)}{r}. \quad (39)$$

Here  $m(r)$  is the Misner-Sharp mass inside radius  $r$ . A classical Schwarzschild solution has  $m(r) = M$ , which leads to a singularity at  $r = 0$ . QNB instead requires that the effective mass

near the center scale as

$$m(r) \sim r^3, \quad (40)$$

so that the density remains finite.

We introduce a saturation law:

$$\frac{dm}{dV} = \rho_c \left(1 - \frac{m}{M}\right)^2, \quad (41)$$

where

$$V = \frac{4\pi r^3}{3}. \quad (42)$$

This equation states that mass-information is locally stored with maximum density  $\rho_c$ , while further storage is screened as the total mass  $M$  is approached.

Solve the equation with  $m(0) = 0$ . Let

$$y = 1 - \frac{m}{M}. \quad (43)$$

Then  $m = M(1 - y)$  and

$$-M \frac{dy}{dV} = \rho_c y^2. \quad (44)$$

Thus

$$\frac{dy}{dV} = -\frac{\rho_c}{M} y^2. \quad (45)$$

With  $y(0) = 1$ ,

$$y(V) = \frac{1}{1 + \rho_c V/M}. \quad (46)$$

Hence

$$m(V) = M \frac{\rho_c V}{M + \rho_c V}. \quad (47)$$

Substituting  $V = 4\pi r^3/3$  gives

$$m(r) = M \frac{r^3}{r^3 + a^3}, \quad (48)$$

with

$$a^3 = \frac{3M}{4\pi\rho_c}. \quad (49)$$

The parameter  $a$  is the QNB core radius: the radius within which the mass  $M$  would fit if stored at the critical information density  $\rho_c$ .

For large  $r$ ,  $r \gg a$ , one has  $m(r) \rightarrow M$ . Therefore  $f(r) \rightarrow 1 - 2M/r$ , the Schwarzschild limit. For small  $r$ ,  $r \ll a$ ,

$$m(r) \approx M \frac{r^3}{a^3}. \quad (50)$$

Thus

$$f(r) \approx 1 - \frac{2M}{a^3} r^2. \quad (51)$$

Define

$$L^2 = \frac{a^3}{2M}. \quad (52)$$

Then

$$f(r) \approx 1 - \frac{r^2}{L^2}. \quad (53)$$

This is a de-Sitter-like core. The classical singularity is replaced by a finite vacuum-energy-like region.

## 8. Finite curvature and energy conditions

A useful test for singularities is the Kretschmann scalar

$$K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \quad (54)$$

For the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega^2 \quad (55)$$

one obtains

$$K = (f'')^2 + 4\left(\frac{f'}{r}\right)^2 + 4\left(\frac{1-f}{r^2}\right)^2. \quad (56)$$

Near the center,  $f(r) \approx 1 - r^2/L^2$ . Hence

$$f'(r) \approx -\frac{2r}{L^2}, \quad f''(r) \approx -\frac{2}{L^2}. \quad (57)$$

Therefore

$$K(0) = \frac{24}{L^4}. \quad (58)$$

This is finite. The classical Schwarzschild singularity,

$$K_{\text{Schw}} = \frac{48M^2}{r^6}, \quad (59)$$

is replaced in QNB by bounded curvature. Restoring  $G$  and  $c$  gives

$$L^2 = \frac{3c^2}{8\pi G\rho_c}. \quad (60)$$

From the mass function,

$$m'(r) = \frac{3Ma^3r^2}{(r^3 + a^3)^2}. \quad (61)$$

The effective density is

$$\rho(r) = \frac{m'(r)}{4\pi r^2} = \frac{3Ma^3}{4\pi(r^3 + a^3)^2}. \quad (62)$$

At  $r = 0$ ,

$$\rho(0) = \frac{3M}{4\pi a^3} = \rho_c. \quad (63)$$

The central density is therefore exactly the critical QNB density.

For the one-function metric  $g_{tt} = -f$ ,  $g_{rr} = f^{-1}$ , the Einstein equations imply an anisotropic effective source with

$$p_r = -\rho \quad (64)$$

and

$$p_t = -\frac{m''(r)}{8\pi r}. \quad (65)$$

For the chosen profile,

$$p_t(r) = \frac{3Ma^3(2r^3 - a^3)}{4\pi(r^3 + a^3)^3}. \quad (66)$$

The weak energy condition requires

$$\rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0. \quad (67)$$

We have

$$\rho \geq 0, \quad (68)$$

$$\rho + p_r = 0, \quad (69)$$

and

$$\rho + p_t = \frac{9Ma^3r^3}{4\pi(r^3 + a^3)^3} \geq 0. \quad (70)$$

The minimal QNB core profile therefore satisfies the weak energy condition.

The strong energy condition requires, among other inequalities,  $\rho + p_r + 2p_t \geq 0$ . Since  $p_r = -\rho$ , this reduces to  $2p_t \geq 0$ . But  $p_t < 0$  for  $2r^3 < a^3$ . The strong energy condition is therefore violated in the core. This is physically unsurprising: singularity resolution requires at least one assumption of the classical singularity theorems to fail. In QNB, this violation is interpreted as effective quantum-information pressure.

The dominant energy condition is more problematic. For  $p_t$ ,

$$\frac{p_t}{\rho} = \frac{2r^3 - a^3}{r^3 + a^3}. \quad (71)$$

For large  $r$ , this approaches 2, so the dominant energy condition is violated in the outer tail of this simple profile. This is not a fatal inconsistency for an effective quantum-gravity model, but it indicates that the saturation profile must be refined in a complete theory.

## 9. Horizon structure of the minimal profile

Horizons occur where  $f(r) = 0$ . With the QNB profile, this means

$$1 - \frac{2Mr^2}{r^3 + a^3} = 0, \quad (72)$$

or

$$r^3 - 2Mr^2 + a^3 = 0. \quad (73)$$

The compactness is

$$\mathcal{C}(r) = \frac{2m(r)}{r} = \frac{2Mr^2}{r^3 + a^3}. \quad (74)$$

This function is zero at  $r = 0$ , approaches  $2M/r$  for large  $r$ , and has a maximum at

$$r = 2^{1/3}a. \quad (75)$$

The maximum is

$$\mathcal{C}_{\max} = \frac{2^{2/3}}{3} \frac{2M}{a}. \quad (76)$$

A black hole exists when  $\mathcal{C}_{\max} \geq 1$ . This requires

$$\frac{2M}{a} \geq \frac{3}{2^{2/3}}. \quad (77)$$

In ordinary units,

$$\frac{r_s}{a} \geq \frac{3}{2^{2/3}}. \quad (78)$$

For sufficiently large mass, two horizons form: an outer horizon  $r_+$  and an inner horizon  $r_-$ . This structure is typical of many regular black-hole models.

The minimal QNB profile resolves the singularity, but it does not yet guarantee a stable inner horizon. A second step is therefore required.

## 10. Stabilization of the inner horizon

Generic regular black holes with two horizons face a serious problem. The inner horizon is often a Cauchy horizon with non-zero surface gravity,

$$\kappa_- \neq 0. \quad (79)$$

In that case, small ingoing and outgoing fluxes are exponentially amplified. The typical growth factor is

$$e^{\kappa_- v}, \quad (80)$$

where  $v$  is an advanced time coordinate. This is the mass-inflation mechanism.

QNB therefore imposes the stability condition

$$\kappa_- = 0. \quad (81)$$

For a static metric with  $ds^2 = -f(r)dt^2 + dr^2/f(r) + r^2d\Omega^2$ ,

$$\kappa_h = \frac{1}{2}|f'(r_h)|. \quad (82)$$

Thus  $\kappa_- = 0$  requires

$$f'(r_-) = 0. \quad (83)$$

A double zero,  $f(r) \sim (r - r_-)^2$ , does not change the sign of  $f$ . We therefore introduce a cubic inner horizon:

$$f(r_-) = 0, \quad f'(r_-) = 0, \quad f''(r_-) = 0, \quad f'''(r_-) \neq 0. \quad (84)$$

Then locally

$$f(r) \sim A(r - r_-)^3. \quad (85)$$

The sign of  $f$  changes, while  $\kappa_- = 0$ . The exponential mass-inflation factor becomes  $e^{\kappa_- v} = 1$ . The classical exponential instability is thereby suppressed.

### 10.1. An explicit inner-extremal QNB metric

We now construct an explicit metric function with four properties: an ordinary outer horizon, an inner-extremal inner horizon, a regular de-Sitter-like core, and Schwarzschild asymptotics at large  $r$ .

Again work in  $G = c = 1$ . Choose parameters

$$0 < r_- < r_+, \quad (86)$$

where  $r_+$  is the outer horizon and  $r_-$  is the QNB transition horizon. Define

$$N(r) = (r - r_+)(r - r_-)^3. \quad (87)$$

Choose

$$D(r) = N(r) + 2Mr^3 + \frac{r_+ r_-^3}{L^2} r^2. \quad (88)$$

Then

$$f_*(r) = \frac{N(r)}{D(r)}. \quad (89)$$

We impose the regularity condition

$$D(r) > 0 \quad \text{for} \quad r \geq 0. \quad (90)$$

This condition prevents additional poles or metric singularities.

For large  $r$ ,

$$N(r) = r^4 - (r_+ + 3r_-)r^3 + O(r^2), \quad (91)$$

and

$$D(r) = r^4 - (r_+ + 3r_- - 2M)r^3 + O(r^2). \quad (92)$$

Therefore

$$f_*(r) = 1 - \frac{2M}{r} + O(r^{-2}). \quad (93)$$

The asymptotic mass is  $M$ , and the exterior geometry is Schwarzschild-like.

At  $r = r_+$ ,  $N(r_+) = 0$ . Since the zero in  $N$  is simple there,  $f'_*(r_+) \neq 0$  for generic parameters. Hence

$$\kappa_+ = \frac{1}{2}|f'_*(r_+)| \neq 0. \quad (94)$$

The outer horizon remains an ordinary black-hole horizon.

At  $r = r_-$ ,  $N(r_-) = 0$ . Since  $N$  has a third-order zero there, and provided  $D(r_-) \neq 0$ ,

$$f_*(r) \sim A(r - r_-)^3. \quad (95)$$

Thus

$$f_*(r_-) = 0, \quad f'_*(r_-) = 0, \quad f''_*(r_-) = 0, \quad (96)$$

and  $f'''_*(r_-) \neq 0$ . Therefore

$$\kappa_- = 0. \quad (97)$$

The inner horizon is inner-extremal.

At  $r = 0$ ,

$$N(0) = r_+ r_-^3, \quad (98)$$

and

$$D(0) = r_+ r_-^3. \quad (99)$$

Thus

$$f_\star(0) = 1. \quad (100)$$

For small  $r$ , the chosen  $D(r)$  gives

$$f_\star(r) = 1 - \frac{r^2}{L^2} + O(r^3). \quad (101)$$

The center is therefore de-Sitter-like, and

$$K(0) = \frac{24}{L^4}. \quad (102)$$

The center is regular. This  $f_\star$ -construction shows that QNB can not only produce a regular core, but also classically stabilize the inner horizon without losing the outer Schwarzschild limit.

## 11. The QNB transition layer

Although  $\kappa_- = 0$  suppresses classical exponential mass inflation, an eternal Cauchy horizon remains physically suspect. Semiclassical calculations suggest that inner-extremal regular black holes may still be unstable once renormalized stress-energy effects are included [16]. QNB therefore interprets the inner horizon not as an exact eternal Cauchy horizon, but as the classical limit of a finite transition layer:

$$r_- - \Delta r < r < r_- + \Delta r. \quad (103)$$

The thickness is of order

$$\Delta r \sim \ell_{\text{QNB}}, \quad (104)$$

where  $\ell_{\text{QNB}}$  is a quantum-information scale, presumably related to  $\ell_P$  or to a larger effective quantum-gravity region.

Inside this layer, the classical geometric description fails. The transition layer has three functions:

1. it prevents an infinite blueshift surface;
2. it couples the black-hole interior geometry to a saturation core;
3. it provides a natural location for bounce or island structures.

The true QNB stability condition is therefore stronger than  $\kappa_- = 0$  alone:

$$\kappa_- = 0, \quad K < K_c, \quad \text{no eternal classical Cauchy horizon.} \quad (105)$$

Here

$$K_c \sim \ell_{\text{QNB}}^{-4}. \quad (106)$$

## 12. Dynamical bounce

A static regular core shows that the singularity can be avoided. A bounce requires a dynamical description of collapsing matter. QNB models the core as a locally homogeneous collapsing region with scale factor  $a_b(\tau)$  and density  $\rho(\tau)$ .

The effective QNB collapse equation is

$$H^2 = \frac{8\pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right), \quad (107)$$

where

$$H = \frac{\dot{a}_b}{a_b}. \quad (108)$$

For  $\rho \ll \rho_c$ , one obtains the ordinary Friedmann-like collapse equation

$$H^2 \approx \frac{8\pi G}{3} \rho. \quad (109)$$

At  $\rho = \rho_c$ ,  $H^2 = 0$ , so  $H = 0$ . The collapse stops.

The associated effective Raychaudhuri equation is

$$\dot{H} = -4\pi G \left( \rho + \frac{p}{c^2} \right) \left( 1 - 2\frac{\rho}{\rho_c} \right). \quad (110)$$

At  $\rho = \rho_c$ ,

$$\dot{H} = +4\pi G \left( \rho_c + \frac{p}{c^2} \right). \quad (111)$$

For matter satisfying  $\rho_c + p/c^2 > 0$ ,  $\dot{H} > 0$ . Thus the system evolves from  $H < 0$  to  $H = 0$  to  $H > 0$ . This is a bounce:

$$\text{collapse} \rightarrow \text{information saturation} \rightarrow \text{expansion}. \quad (112)$$

Planck-star-like models use a comparable idea: quantum-gravity effects halt further collapse at high density, after which the bounce may be short for infalling matter while an external observer sees an extremely long time because of strong time dilation [15].

## 13. New spacetime branch versus traversable portal

The bounce has three possible interpretations.

The most conservative interpretation is that collapse ends in a stable, extremely compact regular core. In that case there is no portal, only singularity resolution.

A second possibility is that, after a very long external time, the bounce appears as a white-hole-like phase. This scenario remains speculative, but it appears in some quantum-gravity-inspired models of black-hole evolution.

The third possibility is that the expansive phase behind the horizon does not return to our

asymptotic exterior region, but forms a new causal branch:

$$\mathcal{M}_1 \rightarrow \mathcal{M}_1 \cup \mathcal{M}_2. \quad (113)$$

In this interpretation, the black hole is not a traversable tunnel through existing space, but a region in which saturated information initiates a new effective spacetime encoding. This is conceptually related to older models in which bounded curvature inside a black hole can lead to a closed or semi-closed new world [7].

QNB therefore does not prove that one can physically travel through a black hole. The more careful claim is:

A QNB interior can be mathematically extended as a new spacetime branch when the bounce behind the horizon expands in a causally disconnected manner.

A traversable portal requires additional conditions, including negative averaged null energy, control over decoherence, and bounded tidal forces. Traversable-wormhole constructions show that negative averaged null energy can temporarily make a wormhole traversable in specific holographic models, but QNB does not derive such a mechanism here [14].

## 14. Information conservation and the Page curve

A non-singular core does not automatically resolve the information paradox. One must also show that the total evolution can be unitary. QNB proposes that the total Hilbert space can be effectively decomposed as

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{rad}} \otimes \mathcal{H}_{\text{ext}} \otimes \mathcal{H}_{\text{core/branch}}. \quad (114)$$

Unitarity requires

$$\rho(t) = U(t)\rho(0)U^\dagger(t). \quad (115)$$

In semiclassical language, the radiation entropy must then display Page-curve-like behavior: it first rises and later falls so that the final state can remain pure.

Modern island calculations use the generalized entropy

$$S_{\text{gen}}(I) = \frac{A(\partial I)}{4G\hbar} + S_{\text{bulk}}(R \cup I), \quad (116)$$

and write the radiation entropy as

$$S(R) = \min_I \text{ext} \left[ \frac{A(\partial I)}{4G\hbar} + S_{\text{bulk}}(R \cup I) \right]. \quad (117)$$

Here  $R$  is the Hawking radiation and  $I$  is a possible island region behind or near the horizon. Replica-wormhole and island results show that such terms can reproduce the Page curve in specific models and reduce the tension between Hawking radiation and unitarity [11, 12].

QNB gives a natural geometric interpretation to the island boundary:

$$\partial I \sim \text{QNB saturation layer}. \quad (118)$$

At early times, the no-island solution dominates:

$$S(R) \approx S_{\text{Hawking}}. \quad (119)$$

After the Page time, the island solution becomes dominant:

$$S(R) \approx \frac{A_{\text{eff}}(t)}{4G\hbar} + S_{\text{bulk}}(R \cup I). \quad (120)$$

Because  $A_{\text{eff}}(t)$  decreases as the black hole evaporates, the radiation entropy can decrease.

The QNB claim is therefore:

The information-saturation layer is the natural candidate for the quantum extremal surface that restores the Page curve.

This is not a complete proof of unitarity, but it makes QNB compatible with the modern island resolution of the information paradox.

## 15. Phenomenological predictions

A theory becomes physically relevant when it predicts measurable deviations from existing models. QNB is designed to remain close to general relativity in the weak-field and ordinary strong-field limits. The observable effects are therefore small.

### 15.1. Exterior-field corrections

For the simple saturation profile,

$$f(r) = 1 - \frac{2Mr^2}{r^3 + a^3}. \quad (121)$$

For  $r \gg a$ ,

$$\frac{1}{r^3 + a^3} = \frac{1}{r^3} \left( 1 - \frac{a^3}{r^3} + O(a^6/r^6) \right). \quad (122)$$

Thus

$$f(r) = 1 - \frac{2M}{r} + \frac{2Ma^3}{r^4} + O(r^{-7}). \quad (123)$$

In ordinary units,

$$f(r) = 1 - \frac{r_s}{r} + \frac{r_s a^3}{r^4} + O(r^{-7}). \quad (124)$$

The first QNB correction therefore falls as  $r^{-4}$ . In ringdown physics, the relevant scale is approximately the photon sphere,

$$r \sim \frac{3}{2}r_s. \quad (125)$$

Hence an order-of-magnitude estimate is

$$\frac{\delta\omega}{\omega} \sim \zeta \left( \frac{a}{r_s} \right)^3, \quad (126)$$

where  $\zeta$  is a dimensionless factor of order unity. If  $a$  is fixed by a Planck-like density, then  $(a/r_s)^3$  is extremely small for astrophysical black holes. This is consistent with the fact that current gravitational-wave tests do not require a convincing deviation from general relativity.

## 15.2. Gravitational-wave echoes

If the QNB transition layer is partially reflective to perturbations, weak echoes may occur after the ordinary ringdown. Place an effective reflective layer at

$$r_0 = r_s(1 + \epsilon), \quad (127)$$

with  $\epsilon \ll 1$ . The echo delay is roughly

$$\Delta t_{\text{echo}} \approx \frac{2r_s}{c} |\ln \epsilon|. \quad (128)$$

If

$$\epsilon \sim \left( \frac{a}{r_s} \right)^p, \quad (129)$$

then

$$\Delta t_{\text{echo}} \approx \frac{2r_s}{c} p \left| \ln \frac{a}{r_s} \right|. \quad (130)$$

The waveform model is then

$$h(t) = h_{\text{GR}}(t) + \sum_{n=1}^{\infty} \mathcal{R}_{\text{QNB}}^n h_{\text{echo}}(t - n\Delta t_{\text{echo}}), \quad (131)$$

where  $\mathcal{R}_{\text{QNB}}$  is the reflection coefficient of the transition layer.

QNB does not necessarily predict strong echoes. If the transition layer is effectively absorbing or remains deeply causally hidden, then  $\mathcal{R}_{\text{QNB}} \approx 0$ . This is important, because analyses of existing data have so far not found a statistically convincing echo population.

## 15.3. Small black holes and primordial populations

For small black holes,  $a/r_s$  becomes larger than for astrophysical black holes. QNB corrections may therefore become relatively more important for hypothetical primordial black holes or in the late phase of evaporation. Planck-star-like models have discussed possible high-energy signals for this reason, although such phenomenological predictions remain highly model-dependent [15].

## 16. Falsifiability

QNB is falsifiable at several levels.

### 16.1. Strong horizon reflectivity

If a concrete QNB version predicts a universal reflection coefficient

$$\mathcal{R}_{\text{QNB}} \gtrsim O(0.1) \quad (132)$$

for astrophysical black holes, but future catalogs find no echoes with sensitivity below that threshold, that version is excluded.

### 16.2. Measurable ringdown deviations

A concrete QNB variant predicts a spectrum

$$\omega_{lmn}^{\text{QNB}} = \omega_{lmn}^{\text{Kerr}} + \delta\omega_{lmn}. \quad (133)$$

If future black-hole spectroscopy measures multiple quasi-normal modes and finds

$$|\delta\omega_{lmn}|/\omega_{lmn} \ll \left(\frac{a}{r_s}\right)^3 \quad (134)$$

for the model's chosen  $a$ , then that parameter choice is excluded.

### 16.3. Inner-horizon instability

If semiclassical calculations show that even with  $\kappa_- = 0$  and a finite transition layer the renormalized stress-energy tensor generically diverges,

$$\langle T_{\mu\nu} \rangle_{\text{ren}} \rightarrow \infty, \quad (135)$$

then the static QNB version fails. The model would survive only as a fully dynamical bounce without a long-lived inner horizon.

### 16.4. No Page-curve compatibility

If the QNB saturation layer cannot provide a consistent quantum extremal surface, or if the Hilbert space of the core/branch inevitably implies information loss, then QNB fails as an information-conserving model.

## 17. Limitations of the model

The model has clear limitations.

First, QNB is not a complete UV-complete theory. The underlying mapping  $g_{\mu\nu} = F_{\mu\nu}[\rho]$  is not derived from first principles. The article specifies an effective theory that has the correct classical limits and avoids singularities.

Second, the choice

$$S_{\text{ind}}(r) = \frac{2\pi E r}{\hbar c} \quad (136)$$

is a saturating weak-field rule based on the Bekenstein bound. A fundamental theory must explain why the induced entropic loading takes precisely this value.

Third, the one-function metric  $g_{tt} = -f$ ,  $g_{rr} = f^{-1}$  is a minimal ansatz. A more realistic model may require two independent functions:

$$ds^2 = -e^{2\Phi(r)} F(r) dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega^2. \quad (137)$$

Fourth, the stabilization of the inner horizon is classical. Semiclassical stability must be tested explicitly through  $\langle T_{\mu\nu} \rangle_{\text{ren}}$ .

Fifth, the new-spacetime-branch interpretation is not necessary. The model requires singularity resolution, but not automatically a traversable portal or physically accessible baby universe.

These limitations do not undermine the model as an effective theoretical framework. Rather, they define the research program.

## 18. Conclusion

We have developed a concrete effective model in which black-hole singularities are replaced by information saturation. The QNB framework yields the following results.

First, from the holographic saturation variable

$$\chi = \frac{4\ell_P^2 S_{\text{ind}}}{A} \quad (138)$$

and the Bekenstein entropic loading of a mass  $M$  on a spherical screen,

$$\chi(r) = \frac{2GM}{c^2 r}. \quad (139)$$

This directly yields the Newtonian potential  $\Phi = -GM/r$ , the force law  $F = GMm/r^2$ , the Schwarzschild horizon  $r = r_s$ , and the standard time-dilation factor

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2GM}{c^2 r}}. \quad (140)$$

Second, a saturation law for the Misner-Sharp mass,

$$\frac{dm}{dV} = \rho_c \left(1 - \frac{m}{M}\right)^2, \quad (141)$$

leads to the profile

$$m(r) = M \frac{r^3}{r^3 + a^3}, \quad (142)$$

with

$$a^3 = \frac{3M}{4\pi\rho_c}. \quad (143)$$

This profile reproduces Schwarzschild at large distance, but gives  $f(r) \approx 1 - r^2/L^2$  near the center, making the Kretschmann scalar finite:

$$K(0) = \frac{24}{L^4}. \quad (144)$$

Third, the main problem of regular black holes, the instability of the inner horizon, is addressed through an inner-extremal QNB condition:

$$\kappa_- = 0. \quad (145)$$

An explicit metric function,

$$f_\star(r) = \frac{(r - r_+)(r - r_-)^3}{(r - r_+)(r - r_-)^3 + 2Mr^3 + \frac{r_+r_-^3}{L^2}r^2}, \quad (146)$$

has an ordinary outer horizon, a cubic inner horizon with zero surface gravity, a regular core, and Schwarzschild asymptotics.

Fourth, the model is compatible with information conservation when the QNB transition layer acts as a natural island region in the generalized entropy. This does not constitute a complete proof of unitarity, but it provides a clear geometric location where the Page-curve correction may arise.

Fifth, QNB has falsifiable, although likely small, phenomenological consequences: ringdown corrections of order

$$\frac{\delta\omega}{\omega} \sim \left(\frac{a}{r_s}\right)^3, \quad (147)$$

possible echoes for non-zero reflectivity of the transition layer, and stronger effects for small or primordial black holes.

The final conclusion is therefore:

QNB is a consistent effective model for non-singular black holes in which information saturation, horizon formation, inner-horizon stabilization, and possible new spacetime branches are connected within a single framework.

The theory is not yet complete as a fundamental theory of quantum gravity. Its scientific value lies in a concrete, falsifiable structure: it reproduces known gravity in the low-saturation limit, avoids central divergences, treats the inner-horizon problem explicitly, and formulates clear points at which future theoretical and observational tests can support or exclude it.

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